

PERFORMANCE MEASURES OF THE MACHINE REPAIR PROBLEM WITH BALKING

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ABSTRACT: In this paper we study a machine repair model with M machines and servers with balking (do not enter the system). We developed some performance measures. We also construct cost information to determine the optimal values of the number of busy servers.

KEYWORDS: Machine repair model, balking, cost, serves

1. INTRODUCTION

Present paper focused on M/M/R machine repair problem with balking, reneging and R unreliable servers. An unreliable server is a server typically and independently subject to unpredictable breakdowns we consider breakdown time and repair time of the server as exponentially distributed. Height considered an M/M/1 queue with customers depending without service. Ancker, Jr.C.J. and Gafarian, Av. (1963) discussed some queuing problems with balking and reneging. In this paper, we consider cost analysis of the M/M/R machine repair problem with balking reneging under server breakdowns discussed by J.C.ke and H. Wang in (1999). Wang, K.H and Wu, J.D. (1995) discussed cost analysis of the M/M/R machine repair problem with spares and two modes of failure. Kleinrock (1975) studied the truncated Poisson queue M/M/C/K/N for machine interference without assumption of balking, reneging or spares.

2. ASSUMPTION

Various assumptions of the model are given as follows

- (i) operating machine fails according to a Poisson process with parameter λ
- (ii) Minimum number of busy servers are denoted by a here we assume $a = 1$
- (iii) Failed machine upon arrival finds $n + k$ failed machines in the system by considering to enter the queue with prob. b_n or balking with prob

$$1 - b_n$$

We define b_n as follows

$$b_n = \begin{cases} 1 & ; n=k=0 \\ \xi & ; n+k>1 \\ 0 & ; \text{other wise} \end{cases}$$

Where ξ is the prob. of a failed machine entering the queue

- (iv) Failed machine sent to a repair it is repaired with repair rate μ
- (v) The server can repair only one failed machine at a time
- (vi) If $a+1$ servers are workings then queue length reach to N .
- (vii) Busy serves are R .
We consider the case $R + N - 1 < M$
- (ix) When there are Y failed machines in the queue and services are completed then the servers that are emptied are cancelled

3. STEADY- STATE EQUATIONS

We present the states by pair (n, k) ; $n = 0, 1, 2, \dots$ $M - R$; $k = 0, 1, \dots, R$ where n is the number of failed machines waiting in the queue and K is the no. of busy servers.

The steady state probability $P_{n,K}$ = prob. That there are n failed machines in the queue and K servers are working by using Markova process the steady state equations for the model are given as follows

$$M\lambda\rho_{00} = \mu\rho_{0,k}; \quad n = k = 0 \quad (1)$$

$$(M - n - 1)\lambda\xi P_{n,1} = \mu P_{n+1,1}; \quad 0 \leq n \leq Y - 1, K = 1 \quad (2)$$

$$(M - n - s)\lambda\xi P_{n,1} = \mu P_{n-1,1} + (M - N)\lambda\xi P_{N-1,1} \\ \gamma \leq n \leq N - 2, K = 1 \quad (3)$$

$$(M - n - s)\lambda\xi P_{n,k} + \kappa\mu P_{\gamma,K} = K\mu P_{n+1,k} + (M - n - k + 1)\lambda\xi P_{N-1,k} \\ Y \leq n \leq N - 2, -2 \leq K \leq R - 1 \quad (4)$$

$$K\mu P_{\gamma,k} = (M - N - K + 2)\lambda\xi P_{N-1,K-1} \quad n = N - 1, 2 \leq K \leq R \quad (5)$$

$$(M - n - R)\lambda\xi P_{n,R} + R\mu P_{\gamma,R} = R\mu P_{n-1,R}, \quad \gamma \leq n \leq N - 2, K = R \quad (6)$$

$$(M - n - R)\lambda\xi P_{n,R} = R\mu P_{n+1,R}, \quad N - 1 \leq n \leq M - R - 1, K = R \quad (7)$$

From equation (2)

$$P_{n+1,1} = \rho\xi(M - n - 1)P_{n,1}; \quad 0 \leq n \leq \gamma - 1 \quad \text{Where } \rho = \frac{\lambda}{\mu} \quad (8)$$

From (8) and using (1)

$$P_{n,1} = \frac{M!}{(M - n - 1)!} \rho^{n+1} \xi^n P_{0,0}; \quad 1 \leq n \leq \gamma \quad (9)$$

From (3) and (9)

$$P_{n,1} = \xi^n \left[\prod_{i=0}^{\infty} (M - i)3 \right] P_{0,0} - \sum_{j=1}^{n-\gamma} (\rho\xi)^j (M - N) \frac{(M - n + j - 2)!}{(M - n - 1)!} P_{N-1,1} \\ \gamma + 1 \leq n \leq N - 1 \quad (10)$$

Equation (10) gives

$$P_{N-1,1} = \frac{\xi^{N-1} \left[\prod_{i=0}^{N-1} (M - i)\rho \right]}{1 + \sum_{j=1}^{N-1-\gamma} (\rho)^j (M - N) \frac{(M - N + j - 1)!}{(M - N)!}} P_{0,0} \quad (11)$$

From equations (4) to (7) we obtain

$$P_{\gamma,K} = \frac{(M - N - K + 2)}{K} P_{\xi} P_{N-1,K-1}; \quad 2 \leq K \leq R, n = N - 1 \quad (12)$$

$$P_{N,R} = \sum_{i=0}^{n-\gamma} \left(\frac{\rho\xi}{R} \right)^i \frac{(M - R - n + k)!}{(M - R - n)!} P_{\gamma,R}, \quad \gamma + 1 \leq n \leq N - 1 \quad (13)$$

$$P_{n,R} = \frac{\prod_{i=N}^n (M - i - R + 1)}{R^{n-N+1}} (\rho\xi)^{n-N+1} P_{N-1,R}, \quad N \leq n \leq M - R \quad (14)$$

From above equation, we obtain $P_{0,0}$ by using

$$P_{0,0} + \sum_{n=0}^{N-1} P_{n,1} + \sum_{K=2}^R \sum_{n=\gamma}^{N-1} P_{n,k} + \sum_{n=N}^{M-R} P_{n,R} = 1 \quad (15)$$

4. PERFORMANCE MEASURES AND COST FUNCTION

L_1 ; The expected no. of failed machines in the system when one server is working

L_k : The expected no, of failed machines in the system when s servers are working, $K = 2, 3, \dots, R-1$

L_R = The expected no. of failed machines in the system when all the servers are working

L : The expected no. of failed machines in the system

L_q : The expected no. of failed machines in the queue.

$E(0)$: The expected no. of operating machines in the system

$E(i)$: Expected no. of idle servers

$E(b)$ Expected no. of busy servers.

$M(a)$: Availability of machine

OU: Operative Utilization

$$L_1 = \sum_{n=0}^{n-1} (n+1)P_{n,1} \quad (16)$$

$$L_5 = \sum_{n=\gamma}^{N-1} (n+K)P_{n,K}, \quad 2 \leq K \leq R-1 \quad (17)$$

$$L_R = \sum_{n=\gamma}^{M-R} (n+R)P_{n,R} \quad (18)$$

$$L = L_1 + \sum_{s=2}^{R-1} L_K + L_R \quad (19)$$

$$E(i) = \sum_{n=0}^{N-1} (R-1)P_{n,1} + \sum_{n=2}^{R-1} \sum_{n=\gamma}^{N-1} (R-K)P_{n,k} \quad (20)$$

$$E(b) = R - E(i) \quad (21)$$

$$B(r) = \sum_{n=0}^{N-1} (M-n-1)\lambda(1-\xi)P_{n,1} + \sum_{n=2}^{R-1} \sum_{n=\gamma}^{N-1} (M-n-K)\lambda(1-\xi)P_{n,s} \\ + \sum_{n=\gamma}^{M-R} (M-n-R)\lambda(1-\xi)P_{n,R} \quad (22)$$

$$\text{Machine availability } M(a) = \frac{E(O)}{M} \quad (23)$$

Where $E(0) = M - L$

And operative utilization

$$OU = \frac{E(b)}{R}$$

Our main objective is to determine the optimum number of busy servers R so as to minimize this function.

We consider the following cost elements

C_1 = Cost per unit time when one failed machine is in the system

C_2 = Cost per unit time when one server is idle

C_3 = Cost per unit time when one server is busy

C_4 = Loss cost per unit time when failed machine walks

Therefore, the total expected cost function per unit time is given as follows

$F(R) = C_1L + C_2E(i) + C_3E(b) + C_4B(r)$ mathematically

$$\min F(R)$$

$$\text{s.t. } 0 < \xi < 1$$

CONCLUSION

In this paper we have discussed M/M/R machine repair problem with balking, reneging & unreliable servers. Our main objective is to minimize the cost and we have developed a cost minimization function.

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